**Classifying Parallelograms**

Rectangles, rhombuses (also called rhombi) and squares are all more specific versions of parallelograms, also called special parallelograms.

|  |  |  |
| --- | --- | --- |
| http://www.ck12.org/flx/show/image/201502261424994727790803_88c4aac2455b2b20ae7970fcdf0147db-201502261424996085148965.pngA quadrilateral is a **rectangle** if and only if it has four right (congruent) angle’s  BCD is a rectangle if and only if angle A \cong \angle B \cong \angle C \cong \angle D. | http://www.ck12.org/flx/show/image/201502261424994727807708_19fa730b03eca076aaa007564d81e92c-201502261424996085354644.pngA quadrilateral is a **rhombus** if and only if it has four congruent sides.  BCD is a rhombus if and only if overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}. | http://www.ck12.org/flx/show/image/201502261424994727837657_6649f973a93e8bf3b6a77f82ee813c9d-201502261424996085534468.pngA quadrilateral is a **square** if and only if it has four right angles and four congruent sides. By definition, ***a square is a rectangle and a rhombus.***  BCD is a square if and only ifangle A \cong \angle B \cong \angle C \cong \angle D and  overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD} |

You can always show that a parallelogram is a rectangle, rhombus, or square by using the definitions of these shapes. There are some additional ways to prove parallelograms are rectangles and rhombuses, shown below:

|  |  |  |
| --- | --- | --- |
| 1.  A parallelogram is a **rectangle** if the diagonals are congruent.  http://www.ck12.org/flx/show/image/201502261424994727856988_db0490e3c3780e9237633d22a06e566b-201502261424996085751792.png BCD is parallelogram. If overline{AC} \cong \overline{BD}, then BCD is also a rectangle. | 2. A parallelogram is a **rhombus** if the diagonals are perpendicular.  http://www.ck12.org/flx/show/image/201502261424994727892666_f315b3037a8386193d65bb27bc29ebaf-201502261424996085996223.png BCD is a parallelogram. If overline{AC} \perp \overline{BD}, then BCD is also a rhombus | http://www.ck12.org/flx/show/image/201502261424994727987073_3f0443cbee0efc34cc183aa145d52ec4-201502261424996086190456.pngA parallelogram is a **rhombus** if the diagonals bisect each angle.  BCD is a parallelogram. If overline{AC} bisects angle BAD  and angle BCD  **and** overline{BD} bisects angle ABC  and angle ADC, then BCD is also a rhombus. |

What if you were given a parallelogram and information about its diagonals? How could you use that information to classify the parallelogram as a rectangle, rhombus, and/or square?

### Examples

#### Example 1

Is a rectangle SOMETIMES, ALWAYS, or NEVER a parallelogram? Explain why.

A rectangle has two sets of parallel sides, so it is ALWAYS a parallelogram.

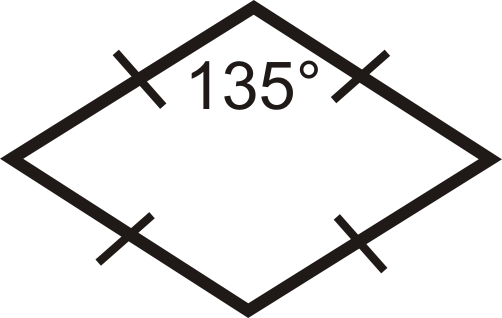
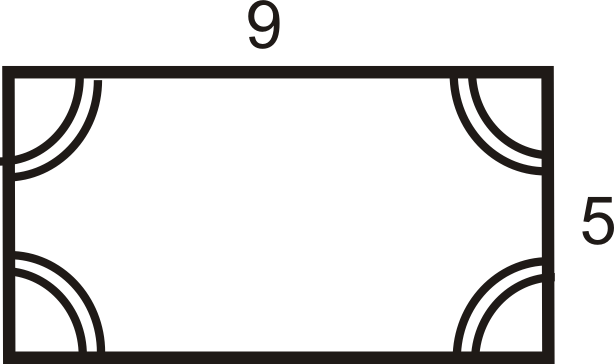
#### Example 2

Is a quadrilateral SOMETIMES, ALWAYS, or NEVER a pentagon? Explain why.

A quadrilateral has four sides, so it will NEVER be a pentagon with five sides.

#### Example 3

What type of parallelogram are the figures below?

For the first figure, all sides are congruent and one angle is 35^\circ, so the angles are not congruent. This is a rhombus.

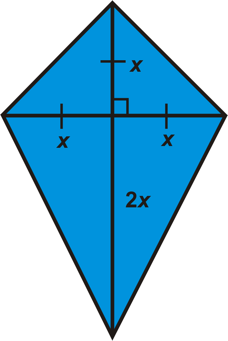
For the second figure, all four angles are congruent but the sides are not. This is a rectangle.

#### Example 4

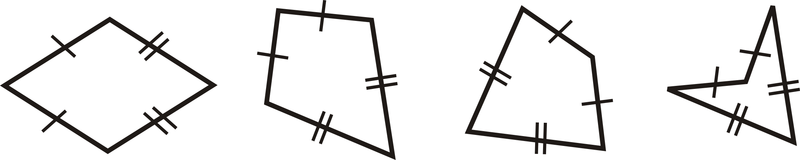
Is a rhombus SOMETIMES, ALWAYS, or NEVER a square? Explain why.

A rhombus has four congruent sides and a square has four congruent sides **and** angles. Therefore, a rhombus is a square when it has congruent angles. This means a rhombus is SOMETIMES a square.

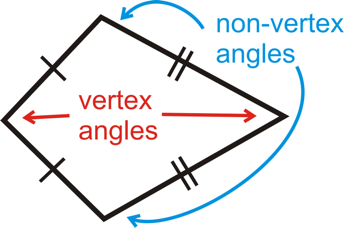
What if you made a traditional kite, seen below, by placing two pieces of wood perpendicular to each other (one bisected by the other)? The typical dimensions are included in the picture. If you have two pieces of wood, 36 inches and 54 inches, determine the values of http://www.ck12.org/flx/math/inline/x and x. Then, determine how large a piece of canvas you would need to make the kite (find the perimeter of the kite).



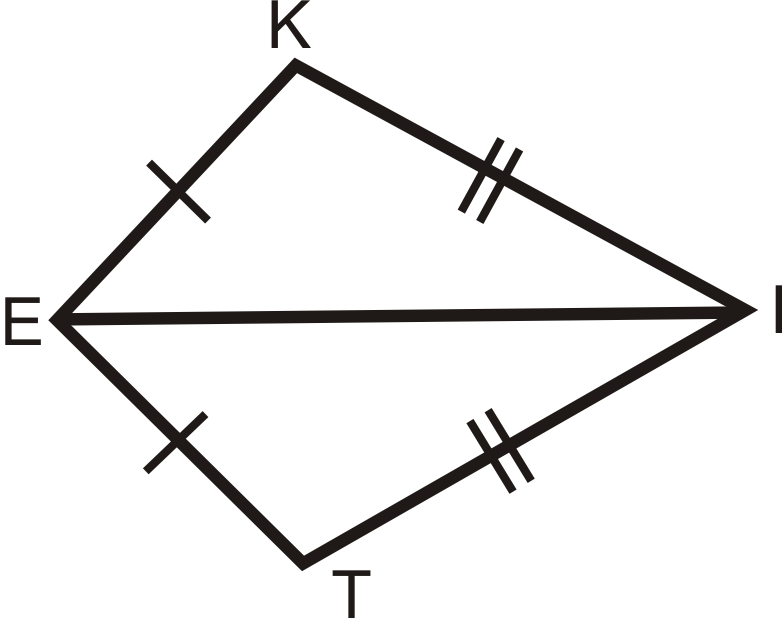
**Kites**

A **kite** is a quadrilateral with two sets of distinct, adjacent congruent sides. A few examples:

From the definition, a kite is the only quadrilateral that we have discussed that could be concave, as with the case of the last kite. If a kite is concave, it is called a ***dart***. The angles between the congruent sides are called ***vertex angles***. The other angles are called ***non-vertex angles***. If we draw the diagonal through the vertex angles, we would have two congruent triangles.



**Theorem:** The non-vertex angles of a kite are congruent.

**Proof:**

Given: ITE with overline{KE} \cong \overline{TE} and overline{KI} \cong \overline{TI}

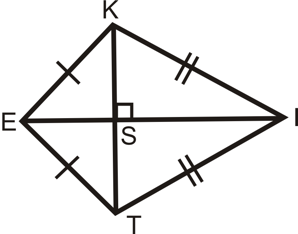
Prove: angle K \cong \angle T

| ***Statement*** | ***Reason*** |
| --- | --- |
| 1. overline{KE} \cong \overline{TE} and overline{KI} \cong \overline{TI} | Given |
| 2. overline{EI} \cong \overline{EI} | Reflexive Property |
| 3. triangle EKI \cong \triangle ETI | SSS |
| 4. angle K \cong \angle T | CPCTC |

**Theorem:** The diagonal through the vertex angles is the angle bisector for both angles.

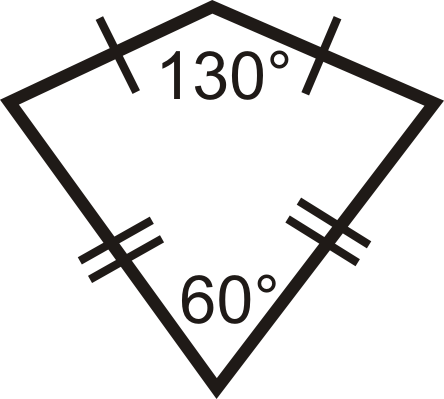
**Kite Diagonals Theorem:** The diagonals of a kite are perpendicular.

To prove that the diagonals are perpendicular, look at triangle KET and triangle KIT. Both of these triangles are isosceles triangles, which means overline{EI} is the perpendicular bisector of overline{KT} (the Isosceles Triangle Theorem). Use this information to help you prove the diagonals are perpendicular in the practice questions.



#### **Measuring Angles**

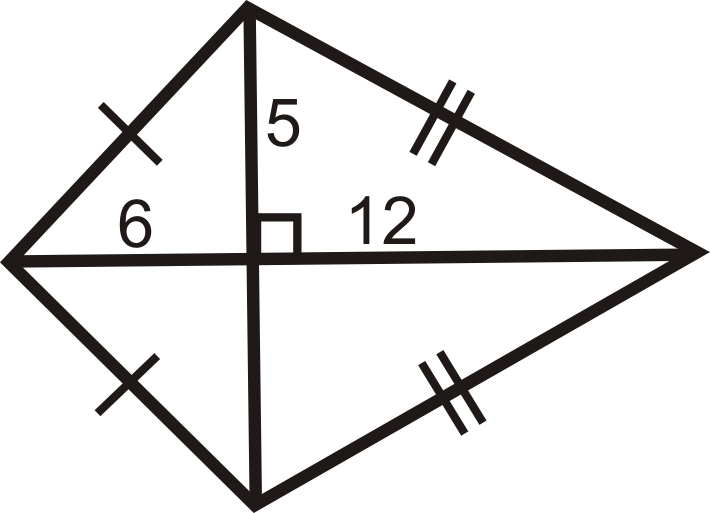
Find the other two angle measures in the kite below.



The two angles left are the non-vertex angles, which are congruent.

#### **Using the Pythagorean Theorem**

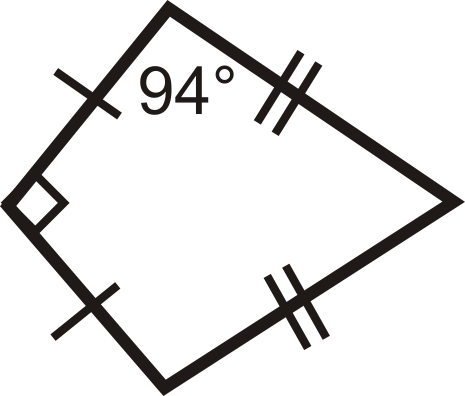
Use the Pythagorean Theorem to find the length of the sides of the kite.



Recall that the Pythagorean Theorem is ^2+b^2=c^2, where http://www.ck12.org/flx/math/inline/c is the hypotenuse. In this kite, the sides are all hypotenuses.

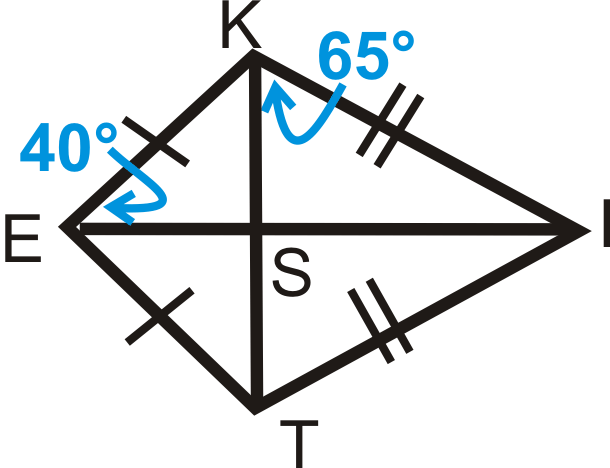
#### **Finding Missing Angle Measures**

Find the other two angle measures in the kite below.

The other non-vertex angle is also 4^\circ. To find the fourth angle, subtract the other three angles from 60^\circ.

0^\circ + 94^\circ + 94^\circ + x & = 360^\circ\\x & = 82

### Examples

ITE is a kite. 

#### Example 1

#### \angle KIS

\angle KIS=25^\circ by the Triangle Sum Theorem (remember that angle KSI is a right angle because the diagonals are perpendicular.)

#### Example 2

#### \angle IST

\angle IST=90^\circ because the diagonals are perpendicular.

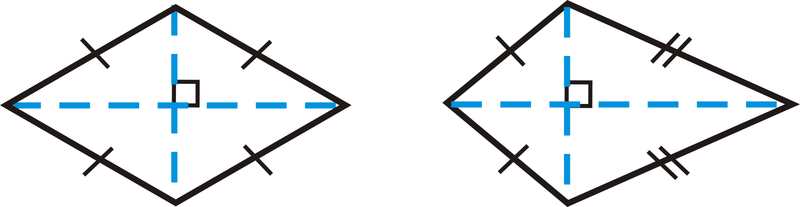
#### Example 3

#### \angle SIT

\angle SIT=25^\circ because it is congruent to angle KIS.

### Area and Perimeter of Rhombuses and Kites

Recall that a **rhombus** is a quadrilateral with four congruent sides and a **kite** is a quadrilateral with distinct adjacent congruent sides. Both of these quadrilaterals have perpendicular diagonals, which is how we are going to find their areas.



Notice that the diagonals divide each quadrilateral into 4 triangles.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Week 29 Guided Notes** | | | | |
| **A parallelogram is a…** | Rectangle if…   |  |  | | --- | --- | |  |  | | | | |
| Square if…   |  | | --- | | 1) | | | | |
| Rhombus if…   |  |  |  | | --- | --- | --- | |  |  |  | | | | |
| In a parallelogram, \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ angles are congruent and \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ angles are supplementary. Since a parallelogram is also a \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ , \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, and \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ then their opposite angles will be \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ and their consecutive angles will be \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_. | | | | |
| **Kite Diagonals Theorem** | |  | | |
| **Non-consecutive angles in a kite therorem** | |  | | |
| **How to find the area** | | | | |
| Of a trapezoid | | | Of a rhombus | Of a kite |
| **How to find the perimeter** | | | | |
| Of a trapezoid | | | Of a rhombus | Of a kite |